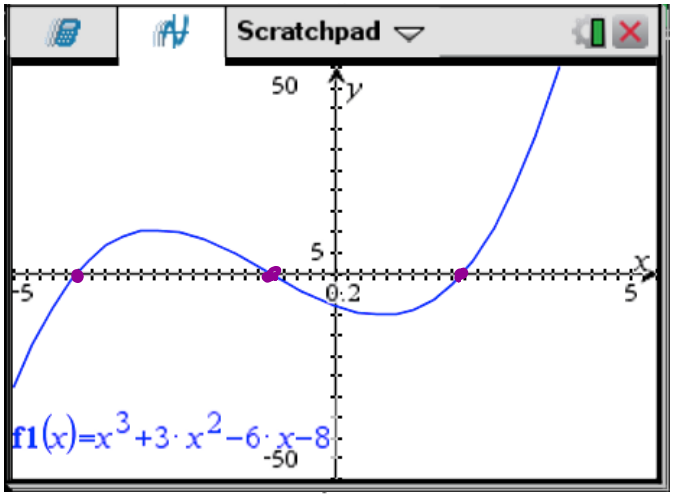
Practice Problems

1. Consider the function $y = x^3 + 3x^2 - 6x - 8$.

   (a) Sketch the function on the axes given. Clearly plot and label each x-intercept.



   (b) Considering your answer to part (a), what values of x are solutions to the equation $x^3 + 3x^2 - 6x - 8 = 0$.

$\{-4, -1, 2\}$

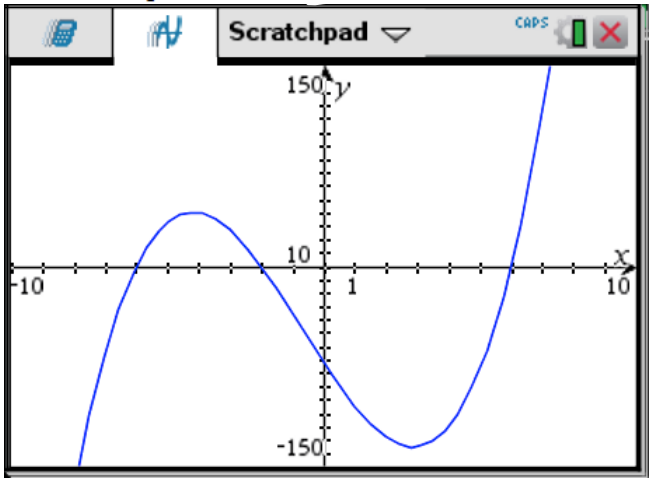
   (c) Based on your answer to part (b), how must the expression $x^3 + 3x^2 - 6x - 8$ factor?

$(x+4)(x+1)(x-2)$

* Menu → Trace → Graph Trace.

2. Consider the cubic function $y = x^3 + 2x^2 - 36x - 72$.

   (a) Find an appropriate y-window for the x-window shown on the axes and sketch the graph. Make the sure the window is sufficiently large to show the two turning points and all intercepts. Clearly label all x-intercepts.



   (b) What are the solutions to the equation $x^3 + 2x^2 - 36x - 72 = 0$?

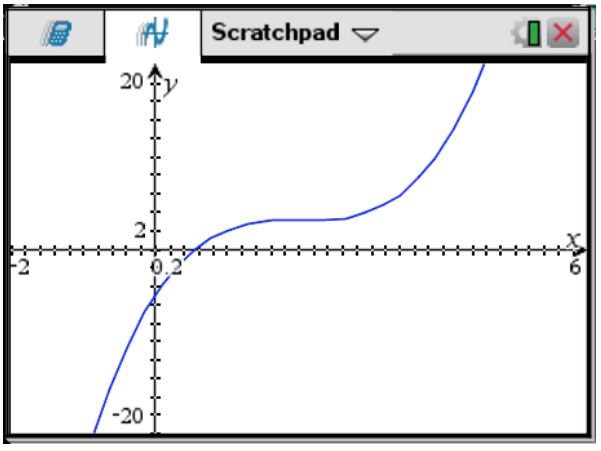
$\{-6, -2, 6\}$

   (c) How does the expression $x^3 + 2x^2 - 36x - 72$ factor?

$(x+6)(x+2)(x-6)$

3. Consider the cubic function given by $y = x^3 - 6x^2 + 12x - 5$.

(a) Sketch a graph of this function on the axes given below.



(b) Considering the graphs of cubics you saw in class and those in problems 1 and 2, what is different about the way this cubic's graph looks compared to the others?

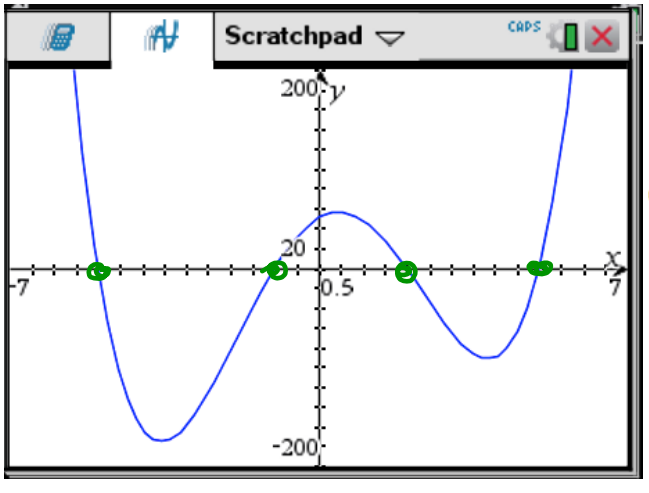
Unlike the other cubics that we saw, this one has no turning points.

The others had two turning points.

This graph is similar to that of $y=x^3$ and in fact is simply a shift of it.

4. Consider the quartic function $y = x^4 - x^3 - 27x^2 + 25x + 50$.

(a) Sketch the graph of this function on the axes given below. Clearly mark all x-intercepts.



(b) Use your graph from part (a) to solve the equation $x^4 - x^3 - 27x^2 + 25x + 50 = 0$.

$\{-5, -1, 2, 5\}$

(c) Considering your answer to (b), how does the expression $x^4 - x^3 - 27x^2 + 25x + 50$ factor?

$(x+5)(x+1)(x-2)(x-5)$

5. In general, how does the number of zeros (or x-intercepts) relate to the highest power of a polynomial? Be specific. Can you make a statement about the minimum number of zeros as well as the maximum?

The maximum number of zeros is equal to the highest power of the polynomial.

- For example, for a quadratic, whose highest power is two, the maximum number of zeros is two. For a quartic, whose highest power is four, the maximum number of zeros is four.

An even powered polynomial can have as few as no zeros whereas an odd powered polynomial, such as a cubic, must have at least one zero because its ends point in opposite directions and thus must cross the x-axis at least once.