Name: ________________________ **Algebra 2/Trig**

12.02 – Radian Angle Measurement

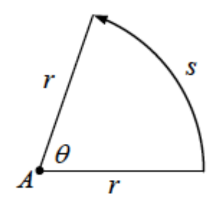
SWBAT: Convert radians to degrees

Just as distance can be measured in inches, feet, miles, centimeters, etcetera, rotations about a point can also be measured in many different ways. Measuring one complete rotation in terms of 360° is somewhat arbitrary. A common unit of angle measurement that is an alternative to degrees is called the radian. It is defined in terms of the arc length of a circle and the circle's radius as shown below:

THE DEFINITION OF A RADIAN

The radian angle, θ, created by a rotation about a point A using a radius of r and passing through an arc length of s is defined as

$$\theta = \frac{s}{r} \quad \text{or equivalently} \quad s = \theta \cdot r$$



***Exercise* #1:** Consider a full rotation around any point in the counter-clockwise (positive) direction.

(a) What is the arc length, s, in terms of the radius of the circle, r, for a full rotation?

(b) Based on the definition above and on your answer to part (a), how many radians are there in one full rotation?

Radians essentially measure the total number of radii (hence the name) that have been traversed about the circumference of a circle in a given rotation. Based on the circumference formula of a circle, thus, there will always be 2π radians in one full rotation.

***Exercise* #2:** Use the formula above to answer each of the following.

(a) Determine the number of radians that the minute hand of a clock passes through if it has a length of 5 inches and its tip travels a total distance of 13 inches.

(b) If a pendulum swings through an angle of 0.55 radians, what distance does its tip travel if it has a length of 8 feet?

Radians are essential in the study of higher-level mathematics and physics and are the angle measurement of choice for the study of calculus. It is important to be able to convert between the angular system of degrees and that of radians. The next exercise will illustrate this process.

***Exercise* #3:** Consider one-half of a full rotation.

(a) What is the angle of rotation in both degrees and in radians?

(b) Using the two equivalent angles of rotation in (a), convert a 30° angle into an equivalent angle in radians.

***Exercise* #4:** Convert each of the following common angles in degrees into radians. Express your answers in terms of pi.

(a) $\theta = 90^\circ$ (b) $\theta = 120^\circ$ (c) $\theta = 225^\circ$

***Exercise* #5:** Convert each of the following common radian angles into degrees.

(a) $\theta = \frac{5\pi}{6}$ (b) $\theta = \frac{3\pi}{2}$ (c) $\theta = \frac{3\pi}{4}$

We should also feel comfortable with the fact that radians do not always have to be in terms of pi, although they often are.

***Exercise* #6:** Convert each of the following radian angles, which aren't in terms of pi, into degrees. Round your answers to the nearest degree.

(a) $\theta = 5.8$ (b) $\theta = 4.2$ (c) $\theta = -2.5$

***Exercise* #7:** An angle drawn in standard position whose radian measure is 2 radians would terminate in which of the following quadrants?

(1) I (3) III

(2) II (4) IV ________

Practice Problems

1. Convert each of the following common degree angles to angles in radians. Express your answers in exact terms of pi.

   (a) 30° (b) 45° (c) 60° (d) 180°

   (e) 300° (f) 135° (g) 270° (h) 330°

2. Convert each of the following angles, given in radians, into degree. Your answers will be integers.

   (a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{2}$ (c) $\frac{11\pi}{4}$ (d) $-\frac{4\pi}{3}$

3. If an angle is drawn in standard position with each of the following radians angles, determine the quadrant its terminal ray lies in. Hint – convert each angle into degrees.

   (a) 4.75 (b) -5.28 (c) 1.65 (d) 7.38

4. Draw a rotation diagram for each of the following radian angles, which are expressed in terms of pi. Then, determine the reference angle for each, also in terms of pi. Think back to how you did this with degrees.

   (a) $\frac{2\pi}{3}$ (b) $\frac{11\pi}{6}$ (c) $\frac{5\pi}{4}$